

FIXED ANGLE SCATTERING AND THE TRANSVERSE STRUCTURE OF HADRONS

GEORGE STERMAN

*C.N. Yang Institute for Theoretical Physics, Stony Brook University
Stony Brook, New York, 11794-3840, USA*

**E-mail: george.sterman@stonybrook.edu*

The perturbative treatment of high-energy fixed-angle hadron-hadron exclusive scattering is reviewed and related to the transverse structure of the proton and other hadrons.

Keywords: Elastic scattering, hadronic wave functions, factorization

1. Introduction

Hadron-hadron exclusive cross sections offer a sensitivity to transverse partonic substructure that is complementary to most photon-induced processes.¹ As we shall see, at high enough energies and momentum transfers, these cross sections probe hadronic Fock states with the minimal numbers of partons, the “valence” states. This talk reviews classic results on hadron-hadron exclusive reactions from this point of view²⁻⁷ and recalls as well some related subsequent work.⁸⁻¹⁰

A full treatment of the transition to asymptotic behavior requires both perturbative resummation and nonperturbative input on the transverse structure of the hadronic valence states. It is not really known to what extent experiments have yet reached a truly asymptotic regime, and pictures involving higher Fock states may offer alternative descriptions. A challenge for the future is to find a unified, perhaps dual, description of exclusive reactions. This short presentation, however, will concentrate on the classic picture based on valence states.

The next section reviews the origin of the basic parton-model “quark counting”^{2,3} predictions for elastic scattering, grounding them in a simple geometric picture, in which transverse structure plays only a passive role. Section 3 introduces the “Landshoff mechanism”,⁴ which reintroduces a

dynamic role for transverse structure, and Sec. 4 shows how an analysis of radiative corrections links the two. In Sec. 5, some intriguing data on wide-angle particle-antiparticle scattering is briefly discussed.

2. Valence states, geometry and quark counting

We begin with the parton model applied to high-energy elastic scattering. In Refs. 2 and 3, elastic scattering is pictured as occurring via time-dilated Fock states with fixed numbers of partons. Suppose all the n_H partons in the valence state for hadron H have comparable momentum fractions x_i . A large coherent momentum transfer $t \sim -Q^2$ is necessary to redirect all these partons into another direction. Such a momentum transfer requires all of the n_H (anti-)quarks to be in a region of area $1/Q^2$ in the Lorentz-contracted wave functions of the colliding hadrons in the center-of-mass frame. This has to be the case for each colliding hadron, and also for the two hadrons that emerge from the scattering. The essential observation is that if the distribution of partons in the transverse direction is random, the likelihood for such a configuration is estimated by $\left(\frac{1}{Q^2} \times \frac{1}{\pi R_H^2}\right)^{n_H-1}$ for each hadron. This geometric picture is illustrated in Figs. 1 and 2.

If the geometric picture determines the overall energy and momentum dependence, then otherwise the amplitude is a function only of the scattering angle. At fixed s/t (that is, fixed c.m. scattering angle) we find,

$$\frac{d\sigma}{dt} = \frac{f(s/t)}{s^2} \left(\frac{m^2}{s}\right)^{\sum_{i=1}^4 (n_i - 1)}. \quad (1)$$

This is the basic parton model result, the ‘quark counting rule’. It clearly picks out the valence state: the cross sections associated with larger numbers of partons are power suppressed, at least as long as we assume that they do not have vanishing momentum fractions.

As the parton model took on a new life in the late 1970’s as a limit of quantum chromodynamics, it was discovered in groundbreaking papers that the corresponding elastic amplitude could be written as^{5,6}

$$\begin{aligned} \mathcal{M}(s, t; h_i) = & \int \prod_{i=1}^4 [dx_i] \phi(x_{m,i}, \lambda_{m,i}, h_i; \mu) \\ & \times M_H \left(\frac{x_{n,i} x_{m,j} p_i \cdot p_j}{\mu^2}; \lambda_{n,i}, h_i \right), \end{aligned} \quad (2)$$

in terms of a calculable hard-scattering amplitude M_H in convolution with factorized and evolved valence (light-cone) wave functions,

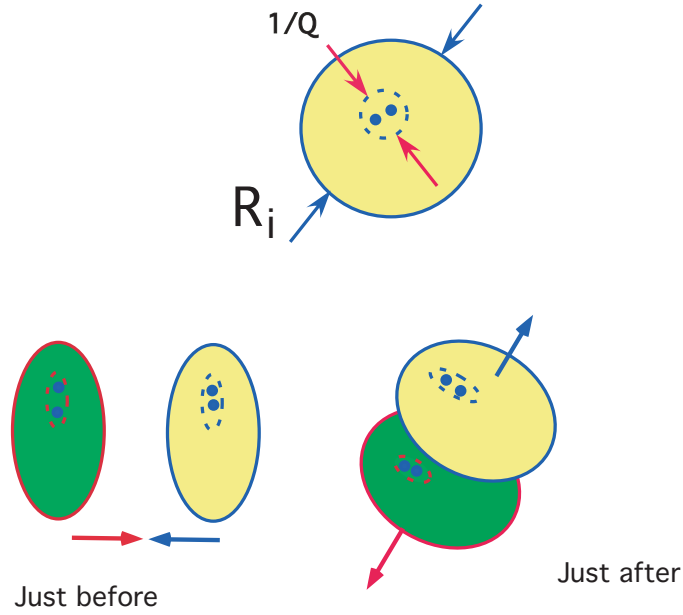


Fig. 1. Valence states before and after an elastic scattering.

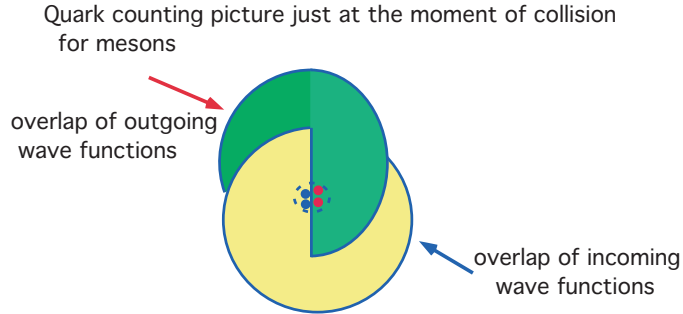


Fig. 2. Incoming and outgoing wave functions at the moment of collision.

$\phi(x_{m,i}, \lambda_{m,i}, h_i; \mu)$, with helicities h_i for hadrons and $\lambda_{n,i}$ for quarks. The mass μ is the factorization scale. The convolution is in terms of the partonic momentum fractions: for baryons,

$$[dx_i] = dx_{1,i} dx_{2,i} dx_{3,i} \delta \left(1 - \sum_{n=1}^3 x_{n,i} \right). \quad (3)$$

So far, the transverse dimensions of colliding hadrons enter only indirectly, in the geometric justification of power behavior.

In principle, all this is straightforward, but our knowledge of the wave functions is not complete, and in any case for nucleon-nucleon scattering there are too many diagrams even at low orders to make a direct calculation practical, at least up to this time.

3. Splitting the hard scattering

The geometric analysis for elastic scattering in the valence state is more flexible than it might at first seem, and there is an alternative picture of the rearrangement of parton momenta, shown in Fig. 3. This is the geometric interpretation of the process identified first by Landshoff.⁴

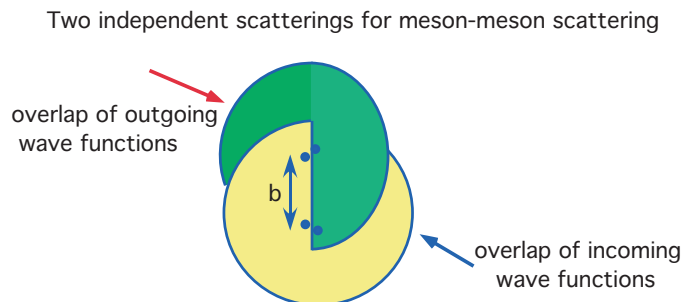


Fig. 3. Representation of independent scatterings.

Figure 3 shows that the single hard scattering of quark counting can in principle be split into as many hard scatterings as there are valence partons in each of the incoming hadrons. (The figure shows two, as for $\pi\pi$ scattering.) Each parton must overlap with an incoming and outgoing wave function. Relative to the single hard scattering picture, however, we no longer require all of the partons to be in the same region of radius $1/Q$. If there are, for example, two hard scatterings, we gain a geometric factor of the transverse size of the hadrons, $R_H Q$ relative to single scattering, with Q the momentum transfer. An enhancement $1/Q \rightarrow R_H$ in the amplitude gives a factor $1/Q^2 \rightarrow R_H^2$ in cross section. In this way, the Landshoff analysis provides an enhancement by a factor $Q^2 \sim s$ for fixed angle pion scattering cross sections and $Q^4 \sim s^2$ for proton-proton fixed angle elastic scattering. More specifically, this modified geometric configuration gives⁴

for pp at fixed angle (*i.e.* fixed s/t)

$$\frac{d\sigma}{dt} = \frac{f(s/t)}{s^2} \left(\frac{1}{s \pi R_H^2} \right)^6, \quad (4)$$

while for $s \gg -t \gg \Lambda_{\text{QCD}}$ it gives

$$\frac{d\sigma}{dt} = \frac{F(s)}{t^2} \left(\frac{1}{t \pi R_H^2} \right)^6. \quad (5)$$

Experimentally, the forward scattering proposal works well at a wide range of center-of-mass energies, but at fixed angles the data appear to follow the original quark counting rules. As I'll now argue, this distinction may be associated with the transverse substructure of hadrons.

4. The return of (approximate) quark counting at wide angles

The scattering of isolated color charges tends to produce radiation in the incoming and outgoing directions. Figure 4 illustrates this effect, superimposed on the parton model template of Fig. 3 for pion scattering. For finite impact parameter b , each of the two scattering processes along the vertical line of overlap involves colored partons. This should lead to the radiation of gluons of wavelength as small as order $1/Q$ and as large as b . Final states without such radiation are suppressed by virtual corrections, unless the impact parameter b is small. On the one hand the Landshoff mechanism is enhanced because there are more ways to produce two hard scatterings than one, but on the other hand it is suppressed because isolated scatterings of colored particles are only rarely elastic. The full amplitude is actually the result of a competition between geometric enhancement and radiative suppression.^{7,11} The resulting balance, and its energy and momentum transfer dependence, was analyzed in Ref. 8.

Impact parameter b is conjugate to $Q = \sqrt{-t}$. As $-t$ increases toward s , radiative corrections force the b 's to $1/\sqrt{s}$ and the quark counting geometric picture should be recovered approximately.^{6,7,12} For color-singlet nucleons, we have as many as three independent scatterings, and this sort of analysis

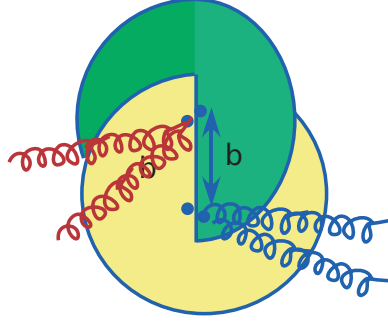


Fig. 4. Schematic representation of radiation that is absent in elastic scattering.

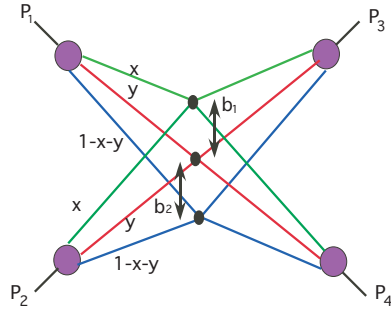


Fig. 5. The Landshoff mechanism in impact parameter space.

leads for baryons to the expression (see Fig. 5)¹⁰

$$\begin{aligned} \mathcal{M}(s, t) = & \frac{N}{stu} \sum_f \int_0^1 \frac{dxdy}{x^2 y^2 (1-x-y)^2} \\ & \times \int db_1 db_2 \text{Tr}_{\text{color}} [U(b_i Q) M^1 M^2 M^3] \\ & \times \prod_{m=1,2,3,4} \Psi_{H_m}(x, y, b_1, b_2), \end{aligned} \quad (6)$$

where N is a normalization factor. The Trace $[U(b_i Q) M^1 M^2 M^3]$ ties color together and includes ϵ_{abc} for colors of three quarks in each external hadron, with possible color exchange in each hard scattering $M^i(x_i p_j)$. The M s are independent of the transverse separations. The matrix U , which depends on both transverse distances b_i , is necessary to incorporate coherent loga-

rithms, left over after the factorization of the wave functions, Ψ .

The wave functions depend on the center-of-mass energy, rather than a factorization scale, as would be the case for parton distributions. This is because they represent the creation or absorption of the partons of the valence state at equal light cone time but fixed transverse separation. In this way, they reflect the amplitude for such a state to exist, without additional degrees of freedom, in a particular frame. They are not gauge invariant, but their product in Eq. (6) is invariant. In fact, it is convenient to construct the wave functions in a physical gauge¹³ and to use their gauge dependence to determine their energy dependence. The pattern is illustrated by the simpler example of pion wave functions, for which one finds an exponentiated form,⁸

$$\Psi(x, b, Q) \sim \Psi_{NP}(x, b) \exp[-s(x, b, Q) - s(1 - x, b, Q)], \quad (7)$$

where the two partons of the valence state carry fractional momenta x and $1 - x$, where $Q \equiv \sqrt{-t}$ and b is the distance between the hard scatterings in Fig. 4. The functions $s(\xi, b, Q)$ provide double-logarithmic suppression whenever the product Qb is large, being defined in terms of universal anomalous dimensions. These are $A(\alpha_s) = C_F(\alpha_s/\pi) + \dots$, which organizes all leading logarithms, and $B(\alpha_s)$, which depends on the details of the factorization procedure,

$$s(x, b, Q) = \int_{C_1/b}^{C_2 x Q} \frac{d\mu}{\mu} \left[4 \ln \left(\frac{C_2 x Q}{\mu} \right) A(\alpha_s(\mu)) + B(\alpha_s(\mu)) \right]. \quad (8)$$

For fixed coupling and small b , one finds that the wave functions have double-logarithmic dependence on the impact parameters,

$$\Psi(x, b, Q) \rightarrow \phi_{asy}(x_j) \exp[-\text{const} \ln^2(1/Qb)]. \quad (9)$$

Assembling the pieces for baryon scattering, this gives the following asymptotic amplitude, with an example of ‘‘Sudakov resummation’’ when $b \gg 1/Q$, which drives the wave functions to their ‘asymptotic’ behaviors^{6,8}

$$\begin{aligned} \mathcal{M}(s, t) = & \frac{N}{stu} \sum_f \int_0^1 \frac{dx_1 dx_2}{x_1^2 x_2^2 (1 - x - y)^2} \prod_{m=1,2,3,4} \phi_{m,asy}(x, y) \\ & \times \int db_1 db_2 \text{Tr}_{\text{color}} [U(b_i Q) M^1 M^2 M^3] \\ & \times e^{-S_1(b_i Q) - S_2(b_i Q) - S_3(b_i Q)}. \end{aligned} \quad (10)$$

At large Q for each scattering, radiation suppression also forces the hard scatterings back together. The S_i are sums of exponential functions like

those in (8), one for each scattering. At moderate $(xQ)^2$, $(yQ)^2$, the amplitude is dominated by the “boundary conditions,” $\Psi_{NP}(x, y, b_i)$ rather than asymptotic behavior. One of the attractive features of this expression is that the original eight integrals over momentum fractions $x_{m,i}$ in Eq. (2) are reduced to only two.

An important feature of Eq. (10) is that the scale of Sudakov suppression is set by the momentum transfer. Varying t at fixed s thus in principle modifies the role of radiative corrections, and as $|t|$ decreases, we anticipate that the cross section makes a transition from s^{-10} to t^{-8} behaviour, as seen in the data.¹⁴ A study of the implications of Eq. (10) for the transverse size of the proton valence state was made in Ref. 10, where it was concluded that the data favor a quite small transverse extent. Fixed-angle scattering on nuclei offers another possible test of the transverse structure of hadrons, via nuclear transparency for small-size color-singlet states.^{15–18}

5. Exchanging quarks

The formalism we’ve described so far is valid asymptotically, but it is not so clear how high the energy and momentum transfer have to be for corrections to these pictures to be negligible. For the multiple scattering picture, in particular, the invariant mass for the “hard scattering” described by the amplitude M_i in Eq. (10) is $x_i^2 s$, typically an order of magnitude smaller than the hadron-level invariants.¹⁹ At the very least, it is clear that the wave functions must vanish sufficiently fast when any of the $x_i \rightarrow 0$ in the valence state. Considerations such as these suggest that at accessible energies, alternative descriptions of exclusive scattering, not necessarily limited to the valence state, should be examined. For the closely related case of form factors, descriptions based on QCD sum rules have shown success.²⁰

With all this in mind, and because of the complexity of the calculations involved, further experimental comparisons of elastic amplitudes between different hadrons may lead to valuable insights. For example, a conceptual contrast was made between gluon and quark exchange processes early on, and has remained of interest.^{21,22} Quark exchange should be highly sensitive to flavor content.

The perturbative factorization formalisms described above can be thought of as involving the exchange of quark degrees of freedom in addition to gluons, and generally these processes contribute to the amplitude in a complicated way. A case of interest, however, is the comparison of $p\bar{p}$ to pp scattering. For pp , there are 2^3 ways of connecting incoming and outgoing quarks compared to only one for $p\bar{p}$. If amplitudes are coherent sums

of these processes, all with a similar weight, then we might expect a ratio of $1/64$ for the elastic scattering cross sections of protons on antiprotons to that for protons on protons.

The relevance of this elegant observation was tested in experiments at Brookhaven National Laboratory's AGS,²³ and ratios of particle-antiparticle to particle-particle seemed roughly consistent with this counting, $R_N = \left. \frac{\frac{d\sigma_{N\bar{N}}}{dt}}{\frac{d\sigma_{NN}}{dt}} \right|_{90^\circ} \sim \frac{1}{40}$. Any successful picture of these exclusive hadronic processes must explain this simple counting result.

Sotiropoulos¹⁰ studied this issue in the Landshoff multiple scattering picture. At first sight, the situation seems promising: at each hard scattering in Fig. 5, quarks may be exchanged (or not) between the participating protons of pp scattering in all possible ways, but for $p\bar{p}$ scattering, the quarks and antiquarks have only one way to get from the initial to the final state. There is a caveat in any pQCD picture, however: we have to reform an antisymmetric color combination of quarks when they are exchanged, a process that requires them to exchange gluons. Sotiropoulos found that the perturbative picture sketched above works qualitatively only with a "color randomization" hypothesis in which the factor $[U(bQ) \prod_i M^i]$ is independent of the flavor flow. He found a ratio of cross sections of $R_p \sim 1/30$ at ninety degrees with color randomization, but only $\sim 1/3$ without it, with a qualitatively successful angular dependence.^{10,24} Randomization is a plausible effect at the moderate BNL energies, $\sqrt{s} \sim 3.5$ GeV², and is easy to picture in the context of quark exchange. This leaves us with a description that is a mixture of perturbative and nonperturbative effects. On the other hand, the Landshoff/Sudakov model with or without randomization gives explicit predictions for angular and helicity dependence. Clearly, a decrease of R_{pp} with energy would be a compelling signal for an emerging role for color.

6. Conclusions

It has been some time since hadron-hadron elastic scattering has held center stage. It remains, however, an important probe of hadron structure, especially for information on pion and nucleon valence states. New facilities may push the frontier in s and t coverage, and clarify essential questions of the role of perturbative and nonperturbative scattering mechanisms. Insights flowing from the development of duality-based pictures of hadronic structure^{25,26} and the increase in our capability to compute multi-parton

scattering amplitudes²⁷ should provide new tools for these fundamental processes.

7. Acknowledgments

This work was supported in part by the National Science Foundation, grant PHY-0653342. I thank Mark Strikman for discussions, and the organizers of “Exclusive Reactions at High Momentum Transfer”, especially Anatoly Radyushkin, Paul Stoler and Christian Weiss, for the invitation to speak at the workshop

References

1. X. D. Ji, J. Phys. G **24**, 1181 (1998) [arXiv:hep-ph/9807358]. A. V. Belitsky and A. V. Radyushkin, Phys. Rept. **418**, 1 (2005) [arXiv:hep-ph/0504030].
2. V. A. Matveev, R. M. Muradian and A. N. Tavkhelidze, *Lett. Nuovo Cim.* **7**, 719 (1973).
3. S. J. Brodsky and G. R. Farrar, *Phys. Rev. D* **11**, 1309 (1975).
4. P. V. Landshoff, *Phys. Rev. D* **10**, 1024 (1974).
5. A. V. Efremov and A. V. Radyushkin, *Phys. Lett. B* **94**, 245 (1980).
6. G. P. Lepage and S. J. Brodsky, *Phys. Rev. D* **22**, 2157 (1980).
7. A. H. Mueller, *Phys. Rept.* **73**, 237 (1981).
8. J. Botts and G. F. Sterman, *Nucl. Phys. B* **325**, 62 (1989).
9. M. G. Sotiropoulos and G. F. Sterman, *Nucl. Phys. B* **425**, 489 (1994) [arXiv:hep-ph/9401237].
10. M. G. Sotiropoulos, *Phys. Rev. D* **54**, 808 (1996) [arXiv:hep-ph/9512397].
11. B. Pire and J. P. Ralston, *Phys. Lett. B* **117**, 233 (1982).
12. P. V. Landshoff and D. J. Pritchard, *Z. Phys. C* **6**, 69 (1980).
13. J. C. Collins and D. E. Soper, *Nucl. Phys. B* **193**, 381 (1981) [Erratum-ibid. B **213**, 545 (1983)].
14. E. Nagy *et al.*, *Nucl. Phys. B* **150**, 221 (1979). W. Faissler *et al.*, *Phys. Rev. D* **23**, 33 (1981).
15. G. Bertsch, S. J. Brodsky, A. S. Goldhaber and J. F. Gunion, *Phys. Rev. Lett.* **47**, 297 (1981); S. J. Brodsky and A. H. Mueller, *Phys. Lett. B* **206**, 685 (1988).
16. P. Jain, B. Pire and J. P. Ralston, *Phys. Rept.* **271**, 67 (1996) [arXiv:hep-ph/9511333].
17. L. Frankfurt, G. A. Miller and M. Strikman, *Phys. Lett. B* **304**, 1 (1993) [arXiv:hep-ph/9305228];
18. A. S. Carroll *et al.*, *Phys. Rev. Lett.* **61**, 1698 (1988); E. M. Aitala *et al.* [E791 Collaboration], *Phys. Rev. Lett.* **86** (2001) 4773 [arXiv:hep-ex/0010044].
19. N. Isgur and C. H. Llewellyn Smith, *Nucl. Phys. B* **317**, 526 (1989).
20. V. A. Nesterenko and A. V. Radyushkin, *Phys. Lett. B* **115**, 410 (1982).
21. J. F. Gunion, S. J. Brodsky and R. Blankenbecler, *Phys. Rev. D* **8**, 287 (1973).

- 22. G. P. Ramsey and D. W. Sivers, *Phys. Rev. D* **45**, 79 (1992). *Phys. Rev. D* **52**, 116 (1995).
- 23. G. C. Blazey *et al.*, *Phys. Rev. Lett.* **55**, 1820 (1985). B. R. Baller *et al.*, *Phys. Rev. Lett.* **60**, 1118 (1988).
C. G. White *et al.*, *Phys. Rev. D* **49**, 58 (1994).
- 24. G. R. Farrar and C. C. Wu, *Nucl. Phys. B* **85**, 50 (1975).
- 25. S. J. Brodsky and G. F. de Teramond, *Phys. Rev. Lett.* **96**, 201601 (2006) [arXiv:hep-ph/0602252].
- 26. H. R. Grigoryan and A. V. Radyushkin, *Phys. Rev. D* **78**, 115008 (2008) [arXiv:0808.1243 [hep-ph]].
- 27. Z. Bern, L. J. Dixon and D. A. Kosower, *Annals Phys.* **322**, 1587 (2007) [arXiv:0704.2798 [hep-ph]].